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Effect of Model Error on Sensor Placement for On-Orbit Modal Identification of Large Space Structures

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A theory is presented for the effect of errors in large space structure prelaunch finite element models on sensor placement for the on-orbit independent identification of a set of selected target modes. The idea of positive net information is introduced. If the net information matrix remains positive definite as sensor locations are deleted from an initial candidate set, the analytical model provides useful information for the identification of the real target modes. If the net information matrix becomes indefinite, the sensor placement analysis based on the prelaunch analytical model will actually detract from the independent identification of the target modes. A bound is presented that can be easily checked after each iteration during which a sensor was deleted to determine positive definiteness of the net information matrix. Numerical examples are used to illustrate the ideas that are presented.

Introduction

PROPOSED large space structures, such as the Space Station illustrated in Fig. 1, will require accurate analytical models for structural analysis, control system design, and simulation. Therefore, testing of the actual structure will have to be performed to identify vibration mode shapes that can be used to correct preliminary analytical models using test-analysis correlation and model-updating techniques.[1-3] The analytical models considered in this paper are generated using finite element methods.

Due to size and flexibility considerations, modal identification must take place on orbit. Minimization of weight and cost will severely limit the sensor resources available. Unlike a ground vibration test in which the structure can be literally plastered with sensors, on-orbit modal identification will require that the relatively small number of sensors are placed in an optimal fashion to identify a small number of targeted mode shapes. Also in contrast with the ground vibration test, sensors cannot be easily moved on orbit. Therefore, extensive prelaunch analysis must be performed to design the sensor configuration and determine if the on-orbit modal identification will be successful. The ultimate goal of a designer is to determine a sensor configuration that will guarantee on-orbit success.

A vast amount of literature[4-12] has been generated dealing with the optimal placement of sensors for identification and control from a control dynamics standpoint. A smaller amount of work[13,14] has been produced considering optimal sensor placement for identification from a structural dynamics point of view. However, only Ref. 15 addresses the important problem of optimal sensor placement from the viewpoint of a structural dynamicist who must use the mode shapes extracted from the test data to update prelaunch models.

Test-analysis correlation techniques require the test modes to be linearly independent or absolutely identifiable.[4] Therefore, the sensors must be placed such that the resulting test modal partitions can be spatially differentiated. Dependent modes will look identical to the correlation analysis. This requirement placed on the sensor configuration is much more stringent than the usual observability[16] required for control purposes and popular modal identification techniques.[17] The approach presented in Ref. 15, called effective independence, ranks candidate sensor locations by their contribution to the linear independence of the analytical target modes. The lowest ranked sensor is eliminated. In an iterative fashion, the initial candidate sensor location set is reduced to the allotted number of sensors. This method tends to maintain the determinant of the Fisher information matrix, resulting in better estimates of the target mode responses.

A common thread running throughout the optimal sensor placement literature is the use of some type of analytical model to perform the placement analysis. None of the work presented in the literature considers the effect of model error in the optimal sensor placement problem. This error can be due to incorrect input parameters, incorrect modeling techniques, unmodeled dynamics, or small nonlinearities. The omission of error effects is especially bothersome in the case of sensor placement for the independent on-orbit identification of selected target modes. The problem arises because the analytical model that is to be updated based on the on-orbit test results is being used to derive the sensor configuration that in turn dictates the modal test data obtained. One must be careful that the tail is not wagging the dog. The error present in the prelaunch finite element model can result in degradation and even failure of the on-orbit modal identification test. This error is especially important in on-orbit identification because a relatively small number of sensors will be used to identify the target modes. The designer of an on-orbit test would like some

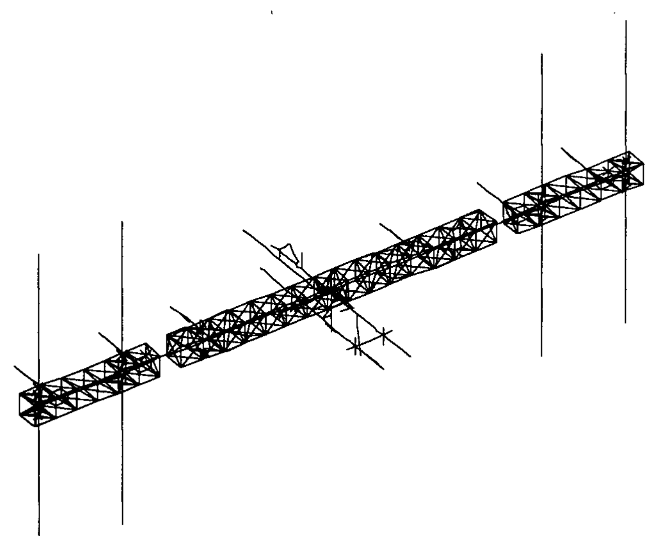

Fig. 1 Space station phase I configuration.

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form of guarantee that the experiment will work based on sensor placement results derived from a prelaunch analytical representation containing modeling error.

This paper presents an error analysis from within the framework of the effective independence method[15] for sensor placement. The analysis provides a lower bound on the number of sensors required to guarantee the spatially independent identification of the true on-orbit target modes using the prelaunch model results. Below this bound, the sensor placement analysis using the prelaunch model actually detracts from the identification of the true target modes.

Error Theory

The error analysis presented in this paper is based on the method of sensor placement called effective independence[15] (EfI). Briefly, the method begins by selecting a set of analytical target modes for identification. This set should include dynamically important modes and any other mode shapes with frequencies very close to the designated important modes. Response from other unimportant mode shapes can be removed from the sensor data based on frequency. Next, a large candidate set of sensor locations is selected that is assumed to contain all the important dynamic information from the target modes. A sensor output equation is assumed in the form

$$u_s = \Phi_{fs} q + N \tag{1}$$

in which Φ_{fs} is the matrix of target modes truncated to the candidate sensor locations, q is a vector containing the target modal displacements, and N represents stationary Gaussian white noise with covariance intensity matrix V. For an efficient unbiased estimator with estimate $\hat{q}$, the covariance matrix of the estimate error is given by

$$P = E[(q-\hat{q})(q-\hat{q})^T] = [\Phi_{fs}^T V^{-1} \Phi_{fs}]^{-1} = Q^{-1} \tag{2}$$

in which E denotes the expected value and Q represents the Fisher information matrix.[18] Maximizing Q results in the minimization of the covariance matrix that provides the best estimate $\hat{q}$. In its present form, the EfI method assumes that the measurement noise is uncorrelated and possesses identical statistical properties for each sensor leading to a diagonal noise covariance matrix. The Fisher information matrix can then be expressed as

$$Q = \frac{1}{\sigma^2}\Phi_{fs}^T \Phi_{fs} = \frac{1}{\sigma^2} A_f \tag{3}$$

where σ^2 is the noise variance in each sensor. Maximization of Q results in the maximization of A_f; therefore, as in Ref. 15, A_f will be referred to as the Fisher information matrix in the remainder of this paper.

The method proceeds by computing the effective independence distribution vector E_D that is the diagonal of the matrix

$$E = \Phi_{fs} A_f^{-1} \Phi_{fs}^T = \Phi_{fs} [\Phi_{fs}^T \Phi_{fs}]^{-1} \Phi_{fs}^T \tag{4}$$

Matrix E can be identified as an orthogonal projector onto the column space of Φ_{fs} with rank equal to the number of target modes. The matrix is therefore idempotent with trace equal to its rank. Terms within E_D thus represent the contribution of each candidate sensor location to the rank and therefore the linear independence of the target mode partitions. Vector E_D is sorted based on magnitude, and the lowest ranked sensor location is deleted from the candidate set. The analysis is repeated, and sensors are removed in an iterative fashion until the desired number of sensors are attained. The method maintains the linear independence of the target mode partitions and tends to maintain the determinant of the Fisher information matrix A_f, which results in superior estimates of modal response from the test data. This approach provides physical insight and a computationally nonintensive method for suboptimally selecting p sensors from a set of n candidate locations.

The effect of prelaunch analytical model error on the sensor placement analysis is studied by assuming that the sensor partition of the analytical target mode shapes can be written as

$$\Phi_{fs} = \Phi_{rs} + \delta_s \tag{5}$$

where Φ_{rs} are the real target mode shapes at the sensor locations and δ_s are the corresponding errors in the prelaunch model target modes. This general form for mode shape error can accommodate errors due to incorrect input parameters, incorrect modeling, unmodeled dynamics, or small nonlinearities. Subtracting δ_s from both sides of Eq. (5) and premultiplying each side by its corresponding transpose yields the relation

$$\Phi_{rs}^T \Phi_{rs} = \Phi_{fs}^T \Phi_{fs} - \Phi_{fs}^T \delta_s - \delta_s^T \Phi_{fs} + \delta_s^T \delta_s \tag{6}$$

Making the following definitions

$$A_r = \Phi_{rs}^T \Phi_{rs}; \qquad \Delta = -\Phi_{fs}^T \delta_s - \delta_s^T \Phi_{fs}; \qquad D = \delta_s^T \delta_s$$

Eq. (6) becomes

$$A_r = A_f + \Delta + D \tag{7}$$

in which A_r represents the information matrix corresponding to the real target mode shapes, Δ is a symmetric matrix representing the uncertainty in the analytical modes, and D is a symmetric matrix representing the information in the mode shape errors δ_s. Information matrix A_f is positive definite by design, matrix Δ is in general indefinite, and without loss of generality, it will be assumed that the unknown error information matrix is also positive definite. The EfI sensor placement strategy will maintain the positive definiteness and determinant of the analytical model information matrix A_f. In reality it is desired to maintain the positive definiteness and determinant of the true information matrix A_r.

In Ref. 15 it was shown that for m target modes there is an m-dimensional ellipsoid, called the absolute identification ellipsoid, associated with the positive definite matrix A_f and given by the relation $x^T A_f^{-1} x = 1$. The ellipsoid has principle axes of length $\lambda_i^{1/2}$, where λ_i are the eigenvalues of A_f, and volume proportional to $|A_f|$. The eigenvectors of A_f span the row space of the target modes Φ_{fs}. The length of a line segment from the center to any point on the ellipsoid represents a measure of the independence of the target modes along the corresponding direction. If one of the eigenvalues λ_i becomes zero as a sensor is deleted from the candidate location set, the ellipsoid collapses and the target modes become dependent. Assuming in the beginning that A_r and D are also positive definite, the real identification ellipsoid can be associated with A_r and an error ellipsoid can be associated with D.

The objective of this work is to determine conditions under which the prelaunch finite element model can be used to make intelligent decisions on sensor placement for identification of the real modes. Subtracting D from both sides of Eq. (7) results in a new matrix I_n given by

$$A_r - D = A_f + \Delta = I_n \tag{8}$$

This new matrix will be called the net information matrix. Examining the left side of Eq. (8), if $A_r - D$ is positive definite (>0), then $A_r > D$, and the real identification ellipsoid defined by A_r contains the error ellipsoid associated with D. In each direction in the identification space, there is more information in the real modes than there is in the analytical mode shape errors. If $A_r - D$ is positive semidefinite (≥ 0), the error ellipsoid now touches the real identification ellipsoid, and finally if $A_r - D$ is indefinite, the error ellipsoid pokes through the real identification ellipsoid.

From Eqs. (7) and (8), and the identified forms of the matrices, it can be seen that $I_n > 0$ is a sufficient condition for the positive definiteness of the real information matrix A_r. Therefore, during the course of an EfI sensor placement analysis, if after each iteration, the net information matrix could be generated and determined to be positive definite, it could be guaranteed that the sensor just eliminated was not vital to the independence of the real target modes. A positive definite net information matrix for any sensor configuration guarantees that the sensors will be able to independently identify the real target modes on orbit.

While the condition $I_n > 0$ is sufficient for the identification of the real target modes, it is not necessary. However, it is proposed that $I_n > 0$ is necessary for the analytical representation to provide any positive or useful information concerning sensor placement for the identification of the real modes. The finite element model is then said to contain positive net information concerning the real target modes Φ_{rs}. Consider the smallest eigenvalue $\lambda_{\min}$ of I_n with corresponding eigenvector $\Psi_{\min}$ possessing unit length. Rearranging Eq. (8) and pre- and postmultiplying by $\Psi_{\min}^T$ and $\Psi_{\min}$, respectively, yields

$$\Psi_{\min}^T A_r \Psi_{\min} = \lambda_{\min} + \Psi_{\min}^T D \Psi_{\min} \tag{9}$$

or

$$a_r = \lambda_{\min} + d \tag{10}$$

Scalar a_r is the square of the Euclidean norm of the vector $\gamma_r = \Phi_{rs}\Psi_{\min}$. The ith term within γ_r is the projection of the ith row of Φ_{rs} onto the unit direction in identification space represented by $\Psi_{\min}$. Therefore, a_r is a measure of the amount of information contained in the real target modes along $\Psi_{\min}$. Likewise, d is a measure of the amount of information contained in δ_s. If I_n is positive definite, $\lambda_{\min} > 0$, the information contained in the analytical mode shape partitions contributes in a positive manner to the information contained in the real target mode partitions along $\Psi_{\min}$. For this case, there is also an ellipsoid associated with the positive definite net information matrix I_n. If I_n becomes positive semidefinite after the deletion of a sensor, $a_r = d$. The net information ellipsoid collapses, and the information contained in the real target modes along $\Psi_{\min}$ is totally due to the error in the corresponding finite element model mode shapes. The analytical representation possesses no knowledge of δ_s; therefore it contributes no information toward the independent identification of the real target modes. If I_n becomes indefinite, $\lambda_{\min} < 0$, the information contained in the model that is used to place sensors for identification of the analytical target models actually detracts from the independent identification of the real target modes. At this point, the analytical representation should no longer be used to place sensors.

A simple example illustrates the point. Consider the case where there are two target modes and an initial candidate set of three sensors. Assume that the real target modes and the corresponding EfI distribution vector are given by

$$\Phi_{rs} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad E_{Dr} = \begin{bmatrix} 1.0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

The EfI sensor ranking indicates that sensor location 1 is vital to the linear independence of the real target modes due to its E_{Dr} value of 1.0. Sensor locations 2 and 3 are of equal importance, and because their E_{Dr} values are less than 1.0, either location can be deleted from the candidate sensor set and the real target modes will still be independently identified. Note that the terms within the vector E_{Dr} add to the number of target modes.

Consider the case in which the analytical target modes and the corresponding EfI distribution are given by

$$\Phi_{fs} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad E_{Df} = \begin{bmatrix} 0.67 \\ 0.67 \\ 0.67 \end{bmatrix}$$

The sensor ranking E_{Df} indicates that each sensor location is of equal importance to the independent identification of the analytical target modes. Any one of the sensors can be eliminated with equal impact on the identification of Φ_{fs}. The finite element model sensor placement analysis therefore provides no information concerning the fact that sensor location 1 is vital to the identification of the real modes. Deletion of a sensor based on the finite element model analysis could lead to the failure of the on-orbit identification of the real target modes. The corresponding net information matrix and eigenpairs are

$$I_n = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \qquad \lambda_1 = 0.0$$

$$\Psi_1 = \begin{bmatrix} 0.71 \\ -0.71 \end{bmatrix} \qquad \lambda_2 = 4.0 \qquad \Psi_2 = \begin{bmatrix} 0.71 \\ 0.71 \end{bmatrix}$$

The net information matrix is indeed positive semidefinite, which is consistent with the previous discussion. The model provides no information along direction Ψ_1. The model does provide positive information concerning the real target modes along direction Ψ_2, which corresponds to the direction of rows 2 and 3 (sensor locations 2 and 3) of the real target modes Φ_{rs}. This result is also consistent with the finite element model sensor placement analysis. No matter which sensor is deleted, the direction Ψ_2 will be represented in the truncated real target modes. This direction will thus also be represented in the real identification ellipsoid.

Next, consider a second case in which the analytical modes and EfI distribution are given by

$$\Phi_{fs} = \begin{bmatrix} 1 & 0.5 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad E_{Df} = \begin{bmatrix} 0.56 \\ 0.56 \\ 0.89 \end{bmatrix}$$

The analysis based on the finite element model now states that the sensor location 3 is the most important whereas locations 1 and 2 are of equal and lesser importance. Sensor 3 would be retained and either sensor 1 or 2 would be deleted. The likelihood that sensor 1, which is vital to the identification of the real target modes, would be deleted has been increased from the previous example. The corresponding net information matrix and eigenvalues are given by

$$I_n = \begin{bmatrix} 2 & 2 \\ 2 & 1.75 \end{bmatrix} \qquad \lambda_1 = -0.13 \qquad \lambda_2 = 3.88$$

The information in the analytical model is now detracting from the identification of the real target modes as indicated by the negative eigenvalue.

Finally, consider a third case in which the analytical modes and EfI distributions are given by

$$\Phi_{fs} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0.5 & 1 \end{bmatrix} \qquad E_{Df} = \begin{bmatrix} 0.89 \\ 0.56 \\ 0.56 \end{bmatrix}$$

The sensor ranking based on the finite element model now ranks sensor 1 as the most important location with sensors 2 and 3 of equal and lesser importance. Sensor 1 is not abso-

Table 1 FEM and real target modal frequencies and corresponding CGM values

Target mode	FEM freq., Hz	Real freq., Hz	CGM
1	4.89	3.83	0.88
2	26.17	21.23	0.85
3	64.04	56.27	0.88
4	88.99	76.24	0.91
5	150.73	124.91	0.76

lutely vital to the identification of the analytical modes, but the overall ranking of the sensors has the same order as the ranking for the real target modes. Sensor 1 will be retained, and either sensor 2 or 3 will be deleted, resulting in the independent identification of the real target modes. The corresponding net information matrix and eigenvalues are given by

$$I_n = \begin{bmatrix} 2.75 & 2 \\ 2 & 2 \end{bmatrix} \qquad \lambda_1 = 0.34 \qquad \lambda_2 = 4.41$$

Positive eigenvalues indicate that I_n is positive definite. Therefore, the model is now providing positive information to the real target mode identification for all directions in identification space. It is important to note that the determinant of the analytical information matrix A_f has the same value of 2.25 for both cases two and three. Thus the total amount of information contained within the finite element model target modes is the same for the two cases. However, the amount of information that is applicable to the identification of the real target modes (net information) did change as the analytical modes were changed.

These simple examples are consistent with the propositions that a positive definite net information matrix I_n is a sufficient condition for the guarantee of the independent identification of the real target modes on orbit and a necessary condition for the intelligent use of the prelaunch analytical representation to place sensors.

Bound Theory

With the error theory presented in the last section, it remains to determine the bounds on a suitable norm of the error δ_s such that the net information matrix is positive definite. Over the past decade, dynamicists working in the area of robust control system design have been concerned with determining bounds on the norm of the perturbation matrix Δ such that if matrix A is stable (all eigenvalues have negative real parts), $A + \Delta$ will remain stable. The results obtained for this stability problem are directly applicable to the determination of the positive definiteness of the perturbed matrix $I_n = A_f + \Delta$ that is being addressed in this paper. Several references[19-23] have considered the case in which A is of a general form and Δ is allowed to be complex and unstructured. However, there is an absence of results specifically addressing the more restricted case of real symmetric A and structured real Δ that is of interest in this work.

For the complex unstructured case, Qiu and Davison[23] present a necessary and sufficient bound for the norm of Δ. Assuming A to be a normal matrix ($A^TA = AA^T$), if Δ is bounded by its L^2 or spectral norm, $A + \Delta$ is stable if and only if

$$\|\Delta\|_2 = \sigma_{\max}(\Delta) < \frac{1}{\|(sI - A)^{-1}\|_\infty} \tag{11}$$

in which $\|\cdot\|_2$ denotes the 2 norm, $\sigma_{\max}$ denotes the maximum singular value, and $\|\cdot\|_\infty$ is the H^∞ norm[23] given by

$$\|(sI - A)^{-1}\|_\infty = \sup_{\omega \geq 0} \{\|(j\omega I - A)^{-1}\|_2\} \tag{12}$$

where sup denotes the supremum and $j = (-1)^{1/2}$. Application of this tight bound to the determination of the positive definiteness of $I_n = A_f + \Delta$ in which both A and Δ are real and symmetric results in the condition

$$\sigma_{\max}(\Delta) < \lambda_{\min}(A_f) \tag{13}$$

where $\lambda_{\min}(\cdot)$ denotes the smallest eigenvalue of A_f. Unfortunately, unlike the general case, while Eq. (13) is sufficient, it is not a necessary condition for the positive definiteness of I_n. This is due to the fact that in the case of interest, the perturbation Δ is structured, i.e.,

$$\Delta = -\Phi_{fs}^T \delta_s - \delta_s^T \Phi_{fs}$$

Only the modal error matrix δ_s itself is unstructured. Therefore Eq. (13) gives a conservative condition on Δ for the positive definiteness of I_n.

Rather than Δ, a condition for the positive definiteness I_n must be derived in terms of a suitable norm of δ_s. The selected measure of modal error size must have physical meaning to the structural dynamicist. For the present case, a condition will be derived in terms of $\|\delta_{\max}\|$, which is the Euclidean norm of the largest modal error vector. That is, $\|\delta_{\max}\| = \max_i \|\delta_{si}\|$ where δ_{si} is the ith column in δ_s. Therefore, to use the condition in Eq. (13), $\sigma_{\max}(\Delta)$ must be related to $\|\delta_{\max}\|$. Using the usual properties of matrix norms and the definition of Δ yields

$$\|\Delta\|_2 \leq \|\Phi_{fs}^T \delta_s\|_2 + \|\delta_s^T \Phi_{fs}\|_2 \leq 2\|\Phi_{fs}\|_2 \|\delta_s\|_2 \tag{14}$$

However,

$$\|\Phi_{fs}\|_2 = \sqrt{\lambda_{\max}(\Phi_{fs}^T \Phi_{fs})} = \sqrt{\lambda_{\max}(A_f)}$$

therefore

$$\|\Delta\|_2 \leq 2\sqrt{\lambda_{\max}(A_f)}\ \|\delta_s\|_2 \tag{15}$$

Considering the modal error information matrix $D = \delta_s^T \delta_s$, where as earlier $\|\delta_s\|_2 = [\lambda_{\max}(D)]^{1/2}$, and using the trace identities

$$tr(D) = \sum_{i=1}^{m} \lambda_i(D) = \sum_{i=1}^{m} \|\delta_{si}\|^2$$

results in the inequality

$$\lambda_{\max}(D) \leq m\|\delta_{\max}\|^2 \tag{16}$$

where m is the number of target modes.

Finally, Eqs. (15) and (16) can be combined to give the desired relation

$$\|\Delta\|_2 \leq 2m^{1/2}\sqrt{\lambda_{\max}(A_f)}\ \|\delta_{\max}\| \tag{17}$$

A sufficient condition for the positive definiteness of the information matrix I_n is thus given by

$$\|\delta_{\max}\| < \frac{1}{2\sqrt{m}} \frac{\sqrt{\lambda_{\max}(A_f)}}{\mathrm{cond}(A_f)} \tag{18}$$

where cond$(\cdot)$ is the condition number in the spectral norm. Assuming a maximum value for target mode error, for example, 10% of the Euclidean norm of the largest analytical target mode, a user can easily compute the right side of Eq. (18) and check for positive definiteness of I_n after each sensor is deleted during the sensor placement analysis. Checking the form of the right side of Eq. (18), it is found to be physically reasonable. It is inversely proportional to the number of target modes, i.e., as the number of target modes increases, more information is needed in the analytical target modal partitions to identify the real target modes. It is also inversely proportional to the condition number of A_f. Orthogonality of the analytical target mode partitions Φ_{fs} promotes the identification of the real target modes.

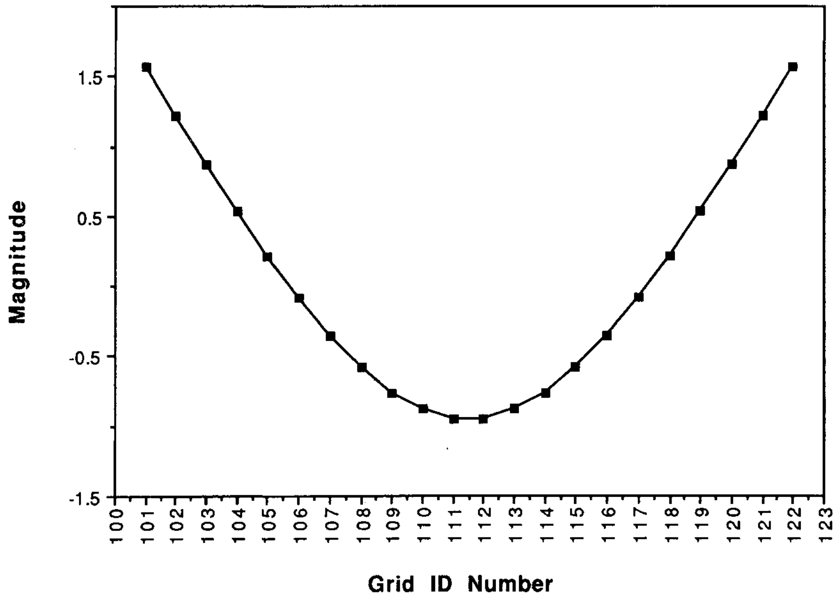

Fig. 2 Simple beam representation of large space structure—first target mode.

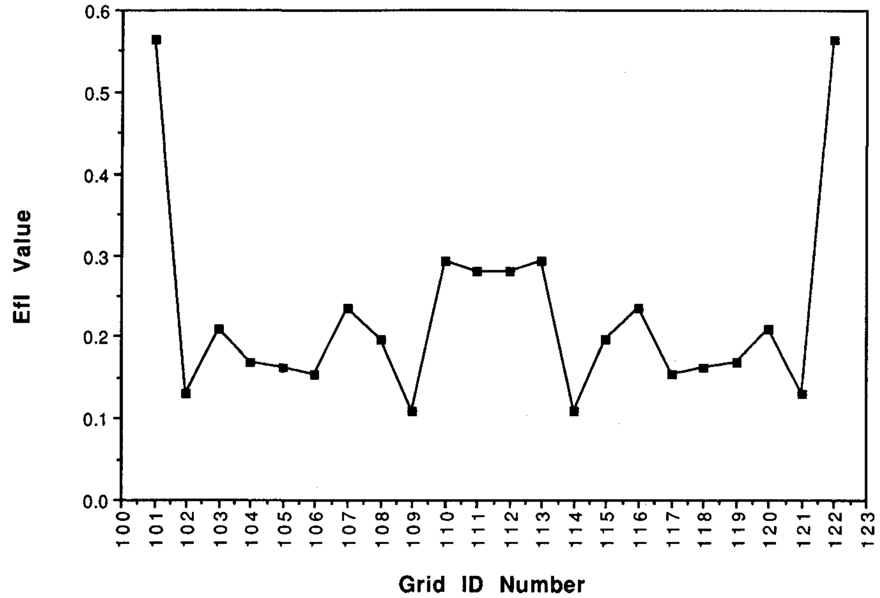

Fig. 3 Effective independence values for 5 FEM target modes and 22 candidate sensor locations.

The drawback of the derived positive definiteness condition given by Eq. (18) is that for many cases it can be very conservative. More research by both the controls and structural dynamics communities on stability conditions for real structured perturbations of real symmetric matrices is needed. However, the derived bound can provide some meaningful insight into the number of sensors required for the guaranteed identification of a set of real target modes given a sensor configuration based on a prelaunch finite element model with an assumed level of error.

Numerical Examples

Simple examples using an elastic beam to simulate a large space structure will demonstrate the ideas put forth in this paper. The finite element model consists of a uniform free-free beam. The actual on-orbit structure is simulated by the same beam with a large concentrated mass at the center representing a significant error in the analytical representation for demonstration purposes. Both models initially contained a transverse displacement and in-plane rotation at each of the 22 grid points along their lengths. The mass and stiffness matrices were statically reduced to just the 22 transverse displacement degrees of freedom. For this example, the 22-degree-of-freedom models will represent the full order analytical and real structure representations.

Eigenvalue solutions for each full order mode produced two rigid body modes and twenty elastic mode shapes. Five of the elastic finite element model mode shapes were selected as target modes. Table 1 lists the analytical target mode frequencies, the corresponding real structure modal frequencies, and the related cross-generalized mass (CGM) values. The cross-generalized mass value for the ith analytical/real target mode pair is computed using the relation

$$CGM_i = \Phi_{fsi}^T M_{FEM} \Phi_{rsi}$$

in which M_{FEM} is the analytical mass matrix. The mode shapes are normalized such that a value of $CGM_i = 1.0$ corresponds to perfect shape correlation between the ith analytical target mode and the corresponding real mode. Values below 0.8

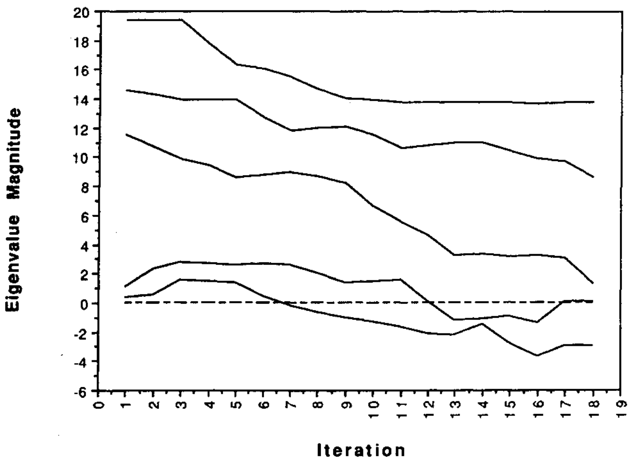

Fig. 4 Eigenvalues of net information matrix I_n vs iteration.

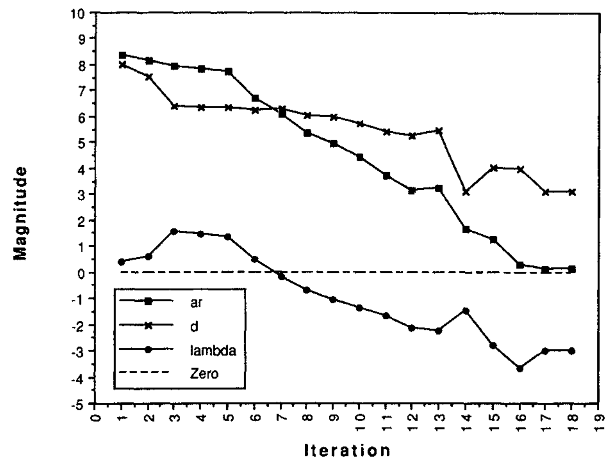

Fig. 5 Information contributions along net information matrix eigenvector $\Psi_{\min}$ vs iteration.

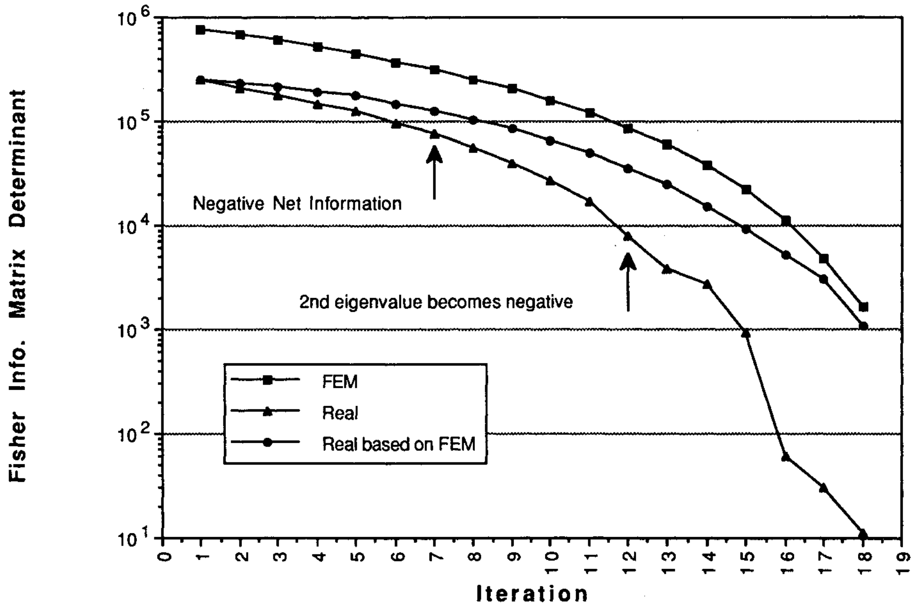

Fig. 6 Fisher information matrix determinants vs iteration.

indicate significant differences between finite element model and real mode shapes. Figure 2 illustrates the first analytical target mode.

The initial candidate sensor location set was assumed to contain all 22 transverse displacement degrees of freedom. The effective independence ranking of each of the initial sensor locations based on the independent identification of the five selected analytical target modes is illustrated in Fig. 3. As expected, the ends of the beam are the most important locations; however, an EfI value of 0.56 indicates that these locations are not absolutely vital to the identification of the target modes. Based on the analytical mode shapes, the EfI method was used to eliminate 17 sensors, one during each iteration, resulting in five suboptimal sensor locations that tend to maintain the determinant of the associated Fisher information matrix.

For this example, the real target modes are known; therefore the net information matrix I_n can be computed at each iteration and checked for positive definiteness. If the net information matrix is positive definite, the finite element model is providing positive or beneficial information to the identification of the real target modes, and thus another sensor can be deleted based on the results of the sensor ranking. Figure 4 illustrates magnitudes of the eigenvalues of the net information matrix for all 17 sensor iterations. At the seventh iteration, after six sensors have been deleted from the initial candidate set, one of the eigenvalues becomes negative, and thus the net information matrix becomes indefinite. A second eigenvalue becomes negative at the 12th iteration.

Beyond the seventh iteration, there exists a direction in the identification space along which the model detracts from the identification of the real target modes. This direction is along the eigenvector $\Psi_{\min}$ corresponding to the minimum eigenvalue as presented in Eqs. (9) and (10). The parameters in Eq. (10) are illustrated in Fig. 5 at each of the 17 iterations for the example sensor placement analysis. Parameter a_r represents the amount of information in the real target modes along $\Psi_{\min}$, d represents the amount of information contained in the analytical mode shape errors, and $\lambda_{\min}$ represents the contribution of the analytical representation. At the seventh iteration, the finite element model contribution becomes negative, and the information within the real target modes becomes less than the information in the modal error vectors.

Figure 6 presents the value of the determinant vs iteration for the finite element model Fisher information matrix, which is a measure of the volume of the corresponding absolute identification ellipsoid. This curve is based on EfI sensor placement using the analytical target modes. Note that the

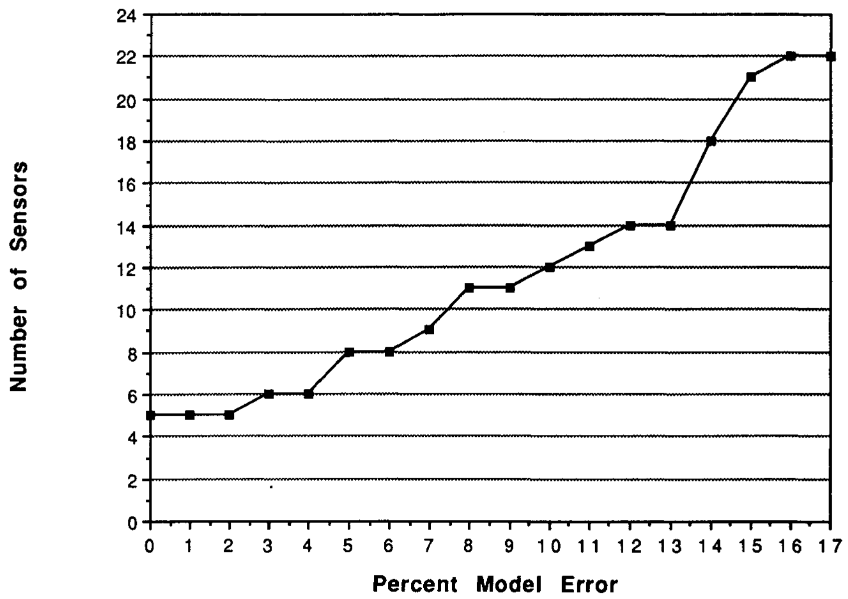

Fig. 7 Number of sensors required for positive definite net information matrix.

method tends to maintain the value of the determinant. The figure also illustrates the value vs iteration of the determinant of the Fisher information matrix corresponding to the real target modes, assuming the sensor placement analysis was based on the real target modes. The third curve in the figure presents the determinant of the real Fisher information matrix vs iteration when the sensor truncation analysis is based on the analytical target modes. This curve represents the actual on-orbit results using a sensor configuration that is derived based on maintaining the independent information in the prelaunch analytical target modes. Note that after the seventh iteration, corresponding to an indefinite net information matrix, the determinant of the real Fisher information matrix falls off rapidly and irregularly. In this regime, the "goodness" of the sensor configuration for the identification of the analytical target modes actually detracts from the independent identification of the real target modes, resulting in large amounts of information loss after a further sensor deletion. This is especially evident after the 12th iteration where a second eigenvalue of I_n has become negative.

Unfortunately, during an actual prelaunch sensor configuration design analysis, the analyst will not have any knowledge of the real target modes. The sensor placement will be based solely on the finite element model. However, the analyst will want to place sensors in sufficient numbers to guarantee the identification of the real target modes on orbit. Assuming a level of modal error based on engineering judgment, Eq. (18) can be used in conjunction with the EfI sensor placement method to determine the iteration at which the prelaunch finite element model can no longer be used for sensor placement. This approach yields the number of sensors required to guarantee on-orbit independent identification based on prelaunch finite element model analysis. Figure 7 illustrates the number of sensors required for on-orbit identification of the real target modes based on the five analytical target modes selected for the uniform beam example assuming varying levels of maximum modal error. The plot indicates that if the length of the maximum error vector is 15.0% of the length of the largest analytical target mode shape, the finite element model will provide no useful information for sensor placement.

Error of 15% is certainly not unexpected in prelaunch analytical representations. However, the analysis used to generate Fig. 7 assumes that all of the modal error vectors are shorter than 15.0% of the longest analytical target mode. Therefore, percentage error corresponding to analytical modes with shorter Euclidean lengths could be greater than 15.0%. Also, as indicated previously, the condition for positive net information given by Eq. (18) can be very conservative in many cases. However, the theory and bound presented in this paper can be used to give sensor configuration designers an idea of the number of sensors required for independent on-orbit modal identification for a given level of model error.

Conclusion

Structural dynamicists will be required to update prelaunch finite element representations of large space structures based on test-analysis correlation techniques that use on-orbit target test mode shapes. To perform this analysis, the test mode shapes must be spatially or linearly independent. A previous work has presented an iterative method called effective independence that suboptimally selects p sensors from a much larger candidate set such that the amount of independent information retained in the analytical target mode partitions is maintained as high as possible. This paper has presented a theory, within the framework of the effective independence method, for the effect of prelaunch model error on the independent on-orbit identification of the true target modes. The idea of a net information matrix has been introduced. If the net information matrix is positive definite, the finite element representation of the target modes provides positive or useful information about the independent identification of the real target mode shapes. The positive definiteness of the net information matrix is a sufficient condition for the guarantee of the independent identification of the real target modes using a sensor configuration derived based on the finite element model. In the regime where net information is indefinite, the goodness of the sensor configuration for the identification of the analytical target modes actually detracts from the independent identification of the real target modes, resulting in large amounts of information loss after further sensor deletions. A conservative bound for assumed model error was also presented, relating the number of target modes and the condition number and the largest eigenvalue of the finite element model Fisher information matrix. Below this bound, the net information matrix is indefinite, and sensor placement based on the finite element model will detract from the identification of the real target modes. If the bound is checked after each iteration during which a sensor is deleted, the number of sensors that are required to guarantee independent identification of the real target modes on orbit can be determined. Even though the bound can be very conservative, the user can still obtain a feel for the number of required sensors for an assumed level of prelaunch model error. As further stability results are obtained from the robust control design community for real symmetric matrices with real structured perturbations, the bound given in this paper can be tightened, providing a more useful tool for on-orbit sensor configuration design.

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